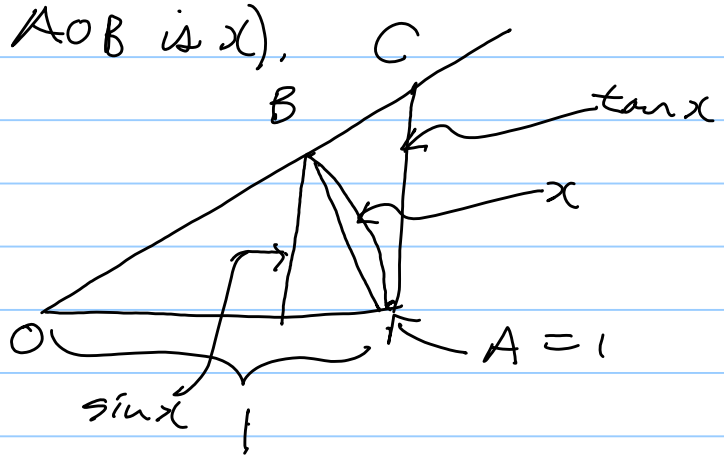
Sine Function

Note Title 10/21/2010

A geometric proof that $\sin x / x \to 1$ as $x \to 0$. (Corrections thanks to Matt Juergl.)

In the figure O is the center of a circle of radius 1, A and B are points on the circle, the x-coordinate of A is 1, the y-coord. of B is $\sin x$, the y-coord. of C is $\tan x$, and the length of arc AB is x (so the radian measure of $\angle AOB$ is x).



$$0 < \text{area}(\Delta OAB) < \text{area}(\text{sector } OAB) < \text{area}(\Delta OAC)$$

$$0 < \frac{1}{2} \cdot 1 \cdot \sin x < \pi \cdot \frac{x}{2\pi} < \frac{1}{2} \cdot 1 \cdot \tan x$$

Hence $0 < \sin x < x < \tan x$,

$$0 < 1 < \frac{x}{\sin x} < \frac{1}{\cos x}.$$

$\cos x \to 1$ as $x \to 0$. Hence $\frac{x}{\sin x} \to 1$ as $x \to 0$.